Accredited

Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics
Sample Question Paper

Date – Morning/Afternoon

Version 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **16** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ \dots\ + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ \dots\ + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ \dots\ + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \ \dots \qquad \left(|x|<1,\ n \in \mathbb{R}\right)$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \tfrac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \textbf{or} \quad \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics
Answer **all** the questions

1 Simplify fully.

(a) $\sqrt{a^3} \times \sqrt{16a}$ **[2]**

1 a) $\sqrt{a^3} \times \sqrt{16a} = \sqrt{16a^4}$
$= 4a^2$

(b) $(4b^6)^{\frac{5}{2}}$ **[2]**

b) $32b^{15}$

2 A curve has equation $y = x^5 - 5x^4$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

2 a) $y = x^5 - 5x^4$
$\frac{dy}{dx} = 5x^4 - 20x^3$
$\frac{d^2y}{dx^2} = 20x^3 - 60x^2$

(b) Verify that the curve has a stationary point when $x = 4$. **[2]**

b) when $x = 4$, $\frac{dy}{dx} = 5(4)^4 - 20(4)^3$
$= 1280 - 1280$
$= 0$

$\frac{dy}{dx} = 0$ hence $x = 4$ is a stationary point

(c) Determine the nature of this stationary point. **[2]**

c) when $x = 4$, $\frac{d^2y}{dx^2} = 20(4)^3 - 60(4)^2$

$= 1280 - 960$

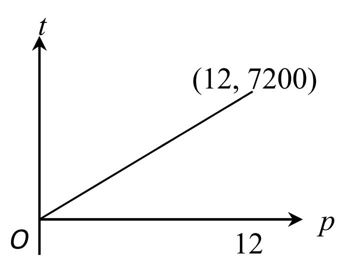
$= 320$

$320 > 0$ hence this stationary point is a minimum

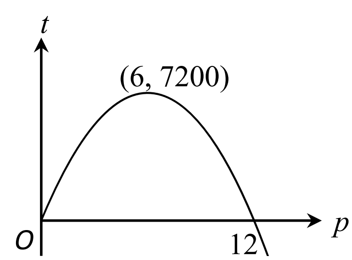
3 A publisher has to choose the price at which to sell a certain new book.
The total profit, £t, that the publisher will make depends on the price, £p.
He decides to use a model that includes the following assumptions.

- If the price is low, many copies will be sold, but the profit on each copy sold will be small, and the total profit will be small.
- If the price is high, the profit on each copy sold will be high, but few copies will be sold, and the total profit will be small.

The graphs below show two possible models.



Model A



Model B

(a) Explain how model A is inconsistent with one of the assumptions given above. **[1]**

3. a) Model A shows a high profit t when p is high

(b) Given that the equation of the curve in model B is quadratic, show that this equation is of the form $t = k(12p - p^2)$ and find the value of the constant k. **[2]**

b) It passes through $(0,0)$ and $(12,0)$, meaning $p - 0$ and $p - 12$ are both roots of the equation

So $t = kp(12 - p)$

we know that when $p = 6$, $t = 7200$

$7200 = k \times 6 \times 6$

$k = 200$

$\therefore t = 200p(12 - p)$

$t = 200(12p - p^2)$

(c) The publisher needs to make a total profit of at least £6400. Use the equation found in part **(b)** to find the range of values within which model B suggests that the price of the book must lie.

[4]

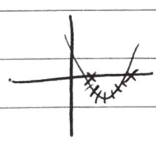
c) $200(12p - p^2) \geqslant 6400$

$2400p - 200p^2 \geqslant 6400$

$0 \geqslant 200p^2 - 2400p + 6400$

$0 \geqslant p^2 - 12p + 32$

$0 \geqslant (p - 4)(p - 8)$



hence $4 \leqslant p \leqslant 8$

Price must be between £4 and £8

(d) Comment briefly on how realistic model B may be in the following cases. **[2]**

- $p = 0$
- $p = 12.1$

d) $p = 0$ is unrealistic because it is giving away the book for free. Therefore t should be negative as they would make a loss
$p = 12.1$ gives a negative value of t. This could be realistic as you may make a loss if p is too high

4 **(a)** Express $\dfrac{1}{(x-1)(x+2)}$ in partial fractions **[2]**

4 a) let $\dfrac{1}{(x-1)(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{x+2}$

$1 = A(x+2) + B(x-1)$

$0 = A + B$
$A = -B$

$1 = 2A - B$
$1 = 2A + A$
$3A = 1$
$A = \dfrac{1}{3} \Rightarrow B = \dfrac{-1}{3}$

$\therefore \dfrac{1}{(x-1)(x+2)} = \dfrac{1}{3(x-1)} - \dfrac{1}{3(x+2)}$

(b) **In this question you must show detailed reasoning.**

Hence find $\int_2^3 \frac{1}{(x-1)(x+2)}\,dx$.

Give your answer in its simplest form. **[5]**

b) $\int_2^3 \frac{1}{(x-1)(x+2)}\,dx = \int_2^3 \frac{1}{3(x-1)} - \frac{1}{3(x+2)}\,dx$

$= \left[\frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+2)\right]_2^3$

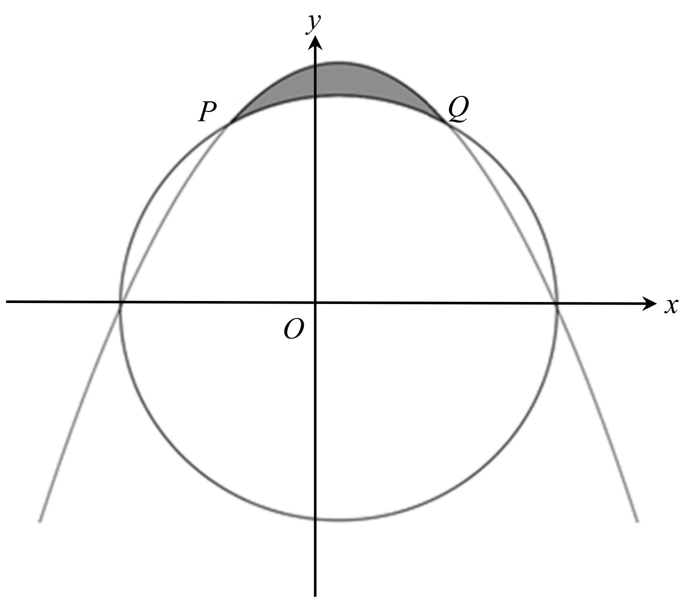
$= \frac{1}{3}\ln(3-1) - \frac{1}{3}\ln(3+2) - \frac{1}{3}\ln(2-1) + \frac{1}{3}\ln(2+2)$

$= \frac{1}{3}\ln 2 - \frac{1}{3}\ln 5 - \frac{1}{3}\ln 1 + \frac{1}{3}\ln 4$

$= \frac{1}{3}\ln\left(\frac{4\times 2}{5}\right)$

$= \frac{1}{3}\ln\left(\frac{8}{5}\right)$

5 The diagram shows the circle with centre O and radius 2, and the parabola $y = \frac{1}{\sqrt{3}}\left(4 - x^2\right)$.



The circle meets the parabola at points P and Q, as shown in the diagram.

(a) Verify that the coordinates of Q are $\left(1, \sqrt{3}\right)$. **[3]**

5 a) The equation of the circle is $x^2 + y^2 = 4$.
Check that $(1, \sqrt{3})$ lies on the circle and the parabola.

Circle : $1^2 + \sqrt{3}^2 = 4$

$1 + 3 = 4$

$4 = 4$ ✓

Parabola : $\sqrt{3} = \frac{1}{\sqrt{3}}\left(4 - 1^2\right)$

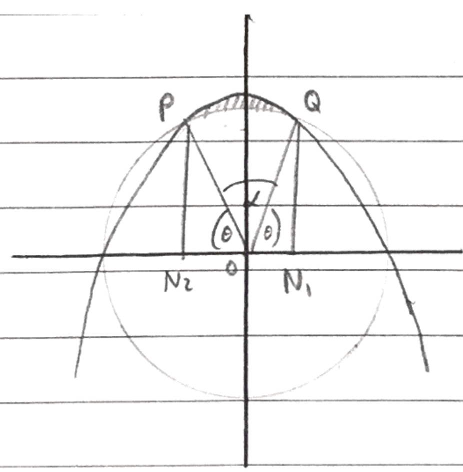
$\sqrt{3} = \frac{1}{\sqrt{3}}(3)$

$\sqrt{3} = \sqrt{3}$ ✓

Hence you know the coordinates of Q must be $(1, \sqrt{3})$

(b) Find the **exact** area of the shaded region enclosed by the arc PQ of the circle and the parabola. **[8]**

b)



N_1 : $(1, 0)$

N_2 : $(-1, 0)$

Area of shape $N_1QPN_2 = \int_{-1}^{1} \frac{1}{\sqrt{3}}(4 - x^2)\,dx$

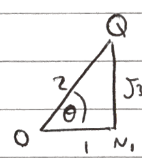
$$= \frac{1}{\sqrt{3}}\left[4x - \frac{1}{3}x^3\right]_{-1}^{1}$$

$$= \frac{1}{\sqrt{3}}\left(4 - \frac{1}{3} + 4 - \frac{1}{3}\right)$$

$$= \frac{22}{3\sqrt{3}}$$

$$= \frac{22\sqrt{3}}{9}$$

Area of sector $POQ = \frac{1}{2}r^2\alpha$



$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} = \alpha$

$$= \frac{1}{2} \times 2^2 \times \left(\frac{\pi}{3}\right)$$

$$= \frac{2}{3}\pi$$

Area of PN_1O = Area of $QN_1O = \frac{\sqrt{3}}{2}$

Shaded area $= \frac{22\sqrt{3}}{9} - \frac{2}{3}\pi - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$$= \frac{13\sqrt{3}}{9} - \frac{2\pi}{3}$$

6 Helga invests £4000 in a savings account.
After t days, her investment is worth £y.
The rate of increase of y is ky, where k is a constant.

(a) Write down a differential equation in terms of t, y and k. **[1]**

6 a) $\frac{dy}{dt} = ky$

(b) Solve your differential equation to find the value of Helga's investment after t days. **[4]**
Give your answer in terms of k and t.

b) $\frac{dy}{dt} = ky$

$\frac{1}{y}\,dy = k\,dt$

$\int \frac{1}{y}\,dy = \int k\,dt$

$\ln y = kt + c$

$y = e^{kt+c}$

$y = Ae^{kt}$

At $t = 0$, $y = 4000$

$4000 = Ae^{0} = A$

$\therefore\ y = 4000e^{kt}$

It is given that $k = \frac{1}{365}\ln\left(1+\frac{r}{100}\right)$ where r % is the rate of interest per annum.

During the first year the rate of interest is 6% per annum.

(c) Find the value of Helga's investment after 90 days. **[2]**

c) when $r = 6$, $k = \frac{1}{365}\ln\left(1 + \frac{6}{100}\right)$

$k = \frac{\ln\left(\frac{106}{100}\right)}{365}$

At $t = 90$, $y = 4000\,e^{90 \times \frac{\ln(1.06)}{365}}$

$y = 4057.8855\ldots$

$y = 4057.89$

After one year (365 days), the rate of interest drops to 5% per annum. **[5]**

(d) Find the total time that it will take for Helga's investment to double in value.

d) when $r = 5$, $k = \frac{1}{365}\ln\left(1 + \frac{5}{100}\right)$

$k = \frac{\ln(1.05)}{365}$

After the first year, her investment is worth

$y = 4000\,e^{365 \times \frac{\ln(1.06)}{365}}$

$y = 4240$

So for the next years, the value of her investment will follow the equation

$y = 4240\,e^{kt}$

because we can use 4240 as the inital amount

She started off with £4000, so we are looking for how long it takes to reach £8000

$8000 = 4240\,e^{kt}$

$$\frac{100}{53} = e^{kt}$$

$$\ln\left(\frac{100}{53}\right) = \frac{\ln(1.05)}{365}t$$

$$t = 4749.53$$

But this is 4749.53 days after the first year ended, so we need to add on the 365 days it took to get the investment to £4240.

$$4749.53 + 365 = 5114.53$$
$$= 5115 \text{ days}$$

Section B: Statistics

Answer **all** the questions

7 **(a)** The heights of English men aged 25 to 34 are normally distributed with mean 178 cm and standard deviation 8 cm.
Three English men aged 25 to 34 are chosen at random.

Find the probability that all three men have a height less than 194 cm. **[3]**

7 a) let X = height of men

$$X \sim N(178, 8^2)$$

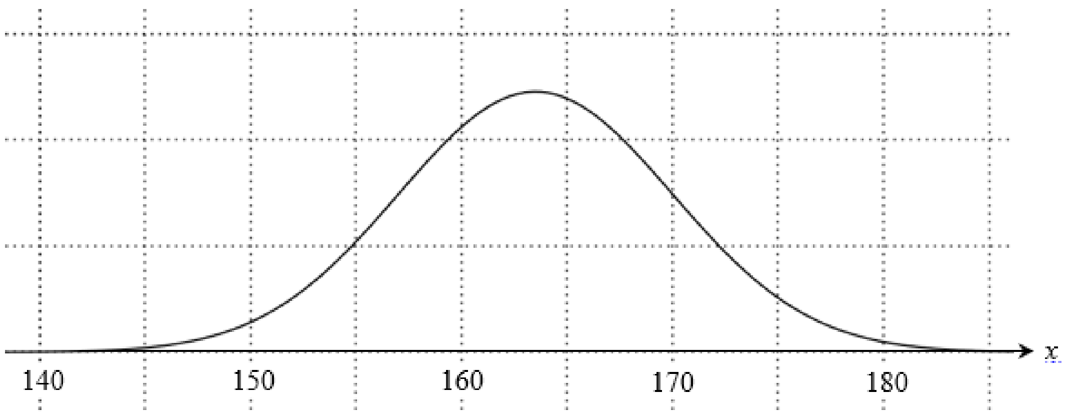
$$P(X < 194) = P\left(\frac{X-178}{8} < \frac{194-178}{8}\right)$$

$$= P(Z < 2)$$
$$= 0.9772$$

This is the probability that one randomly chosen man has a height of less than 194cm

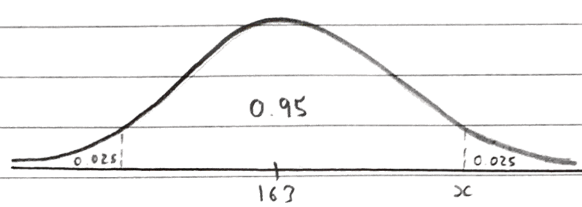
For three men : Probability = 0.9772^3
= 0.9331

(b) The diagram shows the distribution of heights of Scottish women aged 25 to 34.



The distribution is approximately normal. Use the diagram in the Printed Answer Booklet to estimate the standard deviation of these heights, explaining your method. **[3]**

b) 95% of the values lie with 2 standard deviations of the mean
Mean looks approximately 163



$x \approx 177$ from graph
(You are allowed values from 175 to 177)

$x - 163 = 2$ standard deviations
$177 - 163 = 2s$
$14 = 2s$
$s = 7$

8 A market gardener records the masses of a random sample of 100 of this year's crop of plums. The table shows his results.

Mass, m grams	$m < 25$	$25 \le m < 35$	$35 \le m < 45$	$45 \le m < 55$	$55 \le m < 65$	$65 \le m < 75$	$m \ge 75$
Number of plums	0	3	29	36	30	2	0

(a) Explain why the normal distribution might be a reasonable model for this distribution. **[1]**

8 a) The data is symmetrical with the highest values at the centre and lower values at the tails

(b) Find the number of plums in the sample that this model would predict to have masses in the range:

(i) $35 \le m < 45$ **[2]**

weight of plums

$X \sim N(47.5, 10^2)$

i. $P(35 < x < 45)$

$= P\left(\frac{35 - 47.5}{10} < \frac{X - 47.5}{10} < \frac{45 - 47.5}{10}\right)$

$= P(-1.25 < Z < -0.25)$

$= P(Z < -0.25) - P(Z < -1.25)$

$= (1 - 0.5987) - (1 - 0.8944)$

$= 0.2957$

number of plums $= 0.2957 \times 100$

$= 29.57$

$= 30$

(ii) $m < 25$. **[2]**

ii. $P(X < 25)$

$= P\left(\frac{X - 47.5}{10} < \frac{25 - 47.5}{10}\right)$

$= P(Z < -2.25)$

$= 1 - 0.9878$

$= 0.0122$

number of plums $= 0.0122 \times 100$

$= 1.22$

$= \underline{1}$

(c) Use your answers to parts **(b)(i)** and **(b)(ii)** to comment on the suitability of this model. **[1]**

c) 30 is close to the observed value of 29 for $35 \leq m < 45$

1 is close to the observed value of 0 for $m < 25$.

Hence model could be suitable

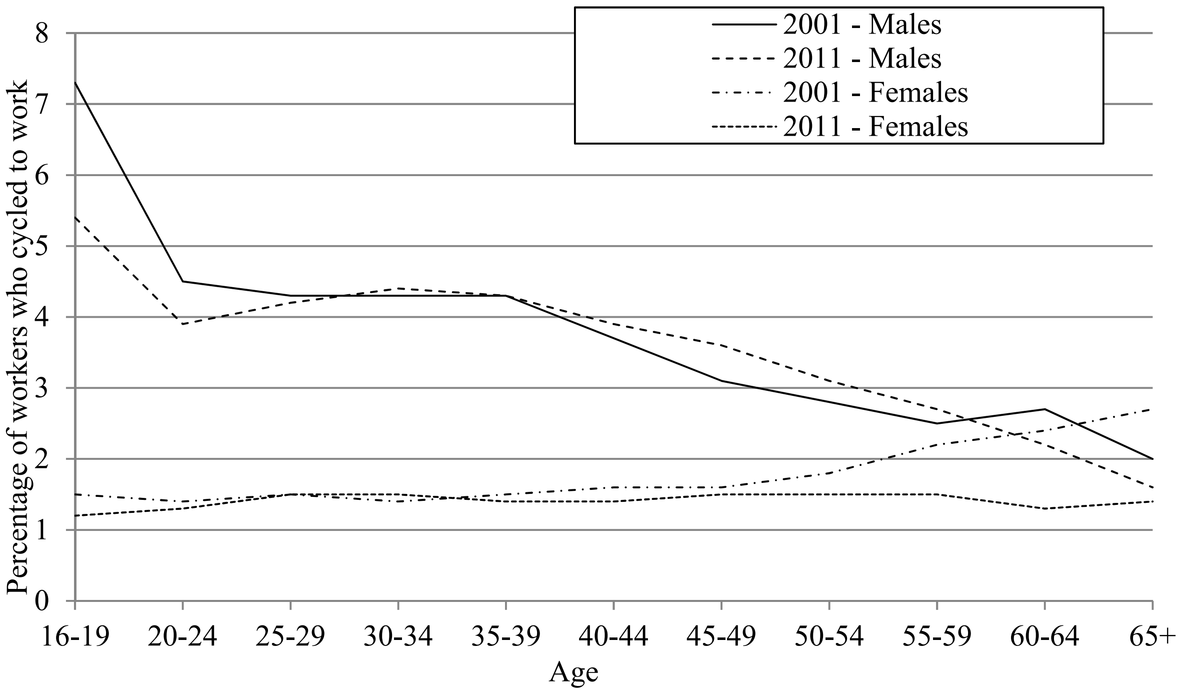
The market gardener plans to use this model to predict the distribution of the masses of next year's crop of plums.

(d) Comment on this plan. **[1]**

d) Next year may be different to this year, meaning the masses of plums may not follow the same distribution

Weather conditions, soil conditions etc may change

9 The diagram below shows some "Cycle to work" data taken from the 2001 and 2011 UK censuses. The diagram shows the percentages, by age group, of male and female workers in England and Wales, excluding London, who cycled to work in 2001 and 2011.



The following questions refer to the workers represented by the graphs in the diagram.

(a) A researcher is going to take a sample of men and a sample of women and ask them whether or not they cycle to work.

Why would it be more important to stratify the sample of men? **[1]**

9 a) The percetage of older men who cycle varies a lot from the percentage of ~~older~~ younger men who cycle. More young men seem to cycle than older men
For women the percentage who cycle remains a lot more consistant between the age groups

A research project followed a randomly chosen large sample of the group of male workers who were aged 30-34 in 2001.

(b) Does the diagram suggest that the proportion of this group who cycled to work has increased or decreased from 2001 to 2011? Justify your answer. **[2]**

b) Decreased
The people in the 30 - 34 group in 2001 are in the 40 - 44 group in 2011
The proportion in the 40 - 44 group is smaller than in the 30 - 34 group

(c) Write down one assumption that you have to make about these workers in order to draw this conclusion. **[1]**

c) The group of people is approximately the same size, with no males ~~joing~~ joining or leaving the group between 2001 and 2011

10 In the past, the time spent in minutes, by customers in a certain library had mean 32.5 and standard deviation 8.2.

Following a change of layout in the library, the mean time spent in the library by a random sample of 50 customers is found to be 34.5 minutes.

Assuming that the standard deviation remains at 8.2, test at the 5% significance level whether the mean time spent by customers in the library has changed. **[7]**

$H_0 : \mu = 32.5$

$H_1 : \mu \neq 32.5$

$\bar{X} \sim N\left(32.5, \frac{8.2^2}{50}\right)$ under the null hypothesis

$$P(\bar{X} > 34.5) = P\left(\frac{\bar{X} - 32.5}{\frac{8.2}{\sqrt{50}}} > \frac{34.5 - 32.5}{\frac{8.2}{\sqrt{50}}}\right)$$

$$= P(Z > 1.725)$$

$$= 1 - 0.9573$$

$$= 0.0427$$

$0.0427 > 0.025$

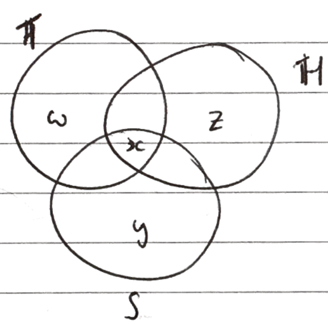
Accept Ho, insufficient evidence that the mean time spent in the library has changed

11 Each of the 30 students in a class plays at least one of squash, hockey and tennis.

- 18 students play squash
- 19 students play hockey
- 17 students play tennis
- 8 students play squash and hockey
- 9 students play hockey and tennis
- 11 students play squash and tennis

(a) Find the number of students who play all three sports. **[3]**

11 a)



First find w, z, y in terms of x

$$y + 11 + 8 - x = 18$$

$$y = x - 1$$

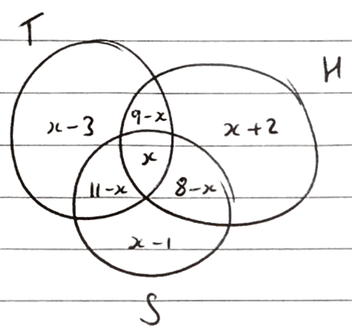
$w + 11 + 9 - x = 17$

$w = x - 3$

$z + 8 + 9 - x = 19$

$z = x + 2$

Now we have



The total of these is equal to 30

$\cancel{x} - 3 + 9 - \cancel{x} + x + 2 + 11 - \cancel{x} + \cancel{x} + 8 - \cancel{x} + x - 1 = 30$

$27 + x = 30$

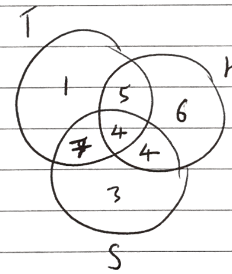
$x = 4$

Hence the number of people who play all three is 4

A student is picked at random from the class.

(b) Given that this student plays squash, find the probability that this student does not play hockey. **[1]**

b)



$$P(H^c \mid S) = \frac{P(H^c \cap S)}{P(S)} = \frac{7+3}{7+3+4+4}$$

$$= \frac{10}{18}$$

$$= \frac{5}{9}$$

Two different students are picked at random from the class, one after the other, without replacement.

(c) Given that the first student plays squash, find the probability that the second student plays hockey. **[4]**

c) There are two cases you need to consider. Either the first person plays hockey as well as squash, or they don't play hockey

$$P(H_2 \mid S_1) = P(H_1 \mid S_1)P(H_2 \mid H_1 \cap S_1) + P(H_1^c \mid S_1)P(H_2 \mid H_1^c \cap S)$$

$$P(H_2 \mid S_1) = \frac{4}{9} \times \frac{18}{29} + \frac{5}{9} \times \frac{19}{29}$$

$$P(H_2 \mid S_1) = \frac{167}{261}$$

12 The table shows information for England and Wales, taken from the UK 2011 census.

Total population	Number of children aged 5-17
56 075 912	8 473 617

A random sample of 10 000 people in another country was chosen in 2011, and the number, m, of children aged 5-17 was noted.
It was found that there was evidence at the 2.5% level that the proportion of children aged 5-17 in the same year was higher than in the UK.
Unfortunately, when the results were recorded the value of m was omitted.

Use an appropriate normal distribution to find an estimate of the smallest possible value of m. **[5]**

12 proportion $= p = \dfrac{8473617}{56075912} = 0.1511$

let X = number of children aged 5-17 in the sample

$X \sim Bin(10{,}000,\ 0.1511)$

mean $= np = 10000 \times 0.1511 = 1511$
standard deviation $= \sqrt{np(1-p)} = \sqrt{10000(0.1511)(1-0.1511)}$
$= \sqrt{1283}$

cv at 2.5% is 1.96

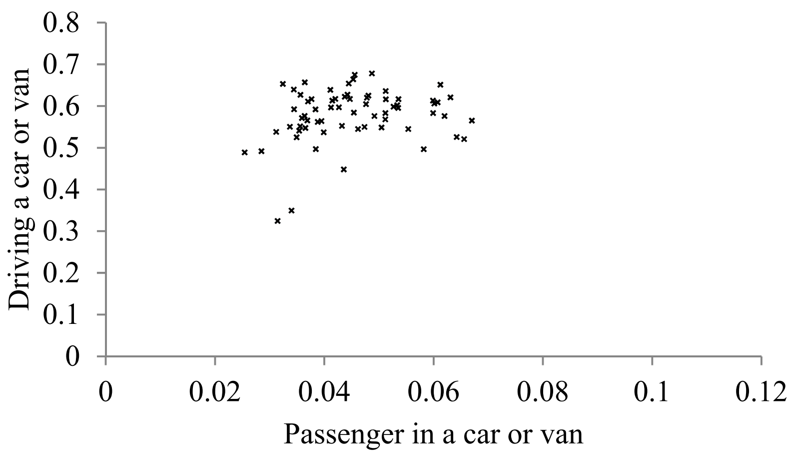
$1511 + 1.96 \times \sqrt{1283} = 1581$

$\therefore$ The minimum value of m is 1581

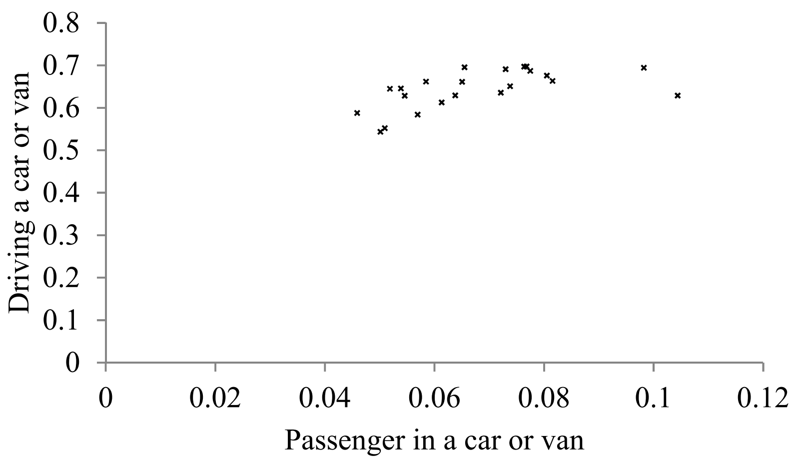
13 The table and the four scatter diagrams below show data taken from the 2011 UK census for four regions. On the scatter diagrams the names have been replaced by letters.
The table shows, for each region, the mean and standard deviation of the proportion of workers in each Local Authority who travel to work by *driving* a car or van and the proportion of workers in each Local Authority who travel to work as a *passenger* in a car or van.
Each scatter diagram shows, for each of the Local Authorities in a particular region, the proportion of workers who travel to work by *driving* a car or van and the proportion of workers who travel to work as a *passenger* in a car or van.

	Driving a car or van		Passenger in a car or van	
	Mean	Standard deviation	Mean	Standard deviation
London	0.257	0.133	0.017	0.008
South East	0.578	0.064	0.045	0.010
South West	0.580	0.084	0.049	0.007
Wales	0.644	0.045	0.068	0.015

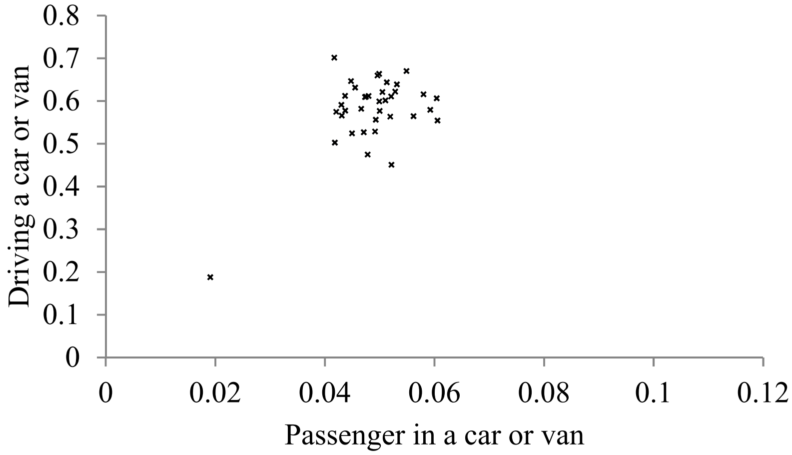
Region A



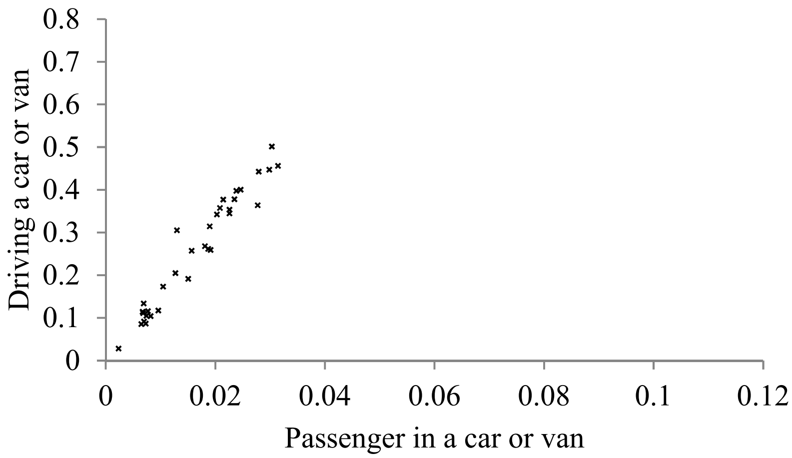
Region B



Region C



Region D



(a) Using the values given in the table, match each region to its corresponding scatter diagram, explaining your reasoning. **[3]**

13 a) Region B is Wales because this is the only graph which has a mean around 0.068. The means on the other graphs are too low for passenger in car or van
Similarly, region D is London because it is the only graph that could have a mean as low as 0.017 for passenger
Regions A and C are similar, but region C has a larger spread of driving because of its outlier. This means that C is the South West because the value of the standard deviation here is higher than it is for the South East
Therefore region A is the South East

(b) Steven claims that the outlier in the scatter diagram for Region C consists of a group of small islands.

Explain whether or not the data given above support his claim. **[1]**

b) No, it just shows that this particular local authority has a lower proportion of people travelling to work by car or van

(c) One of the Local Authorities in Region B consists of a single large island.

Explain whether or not you would expect this Local Authority to appear as an outlier in the scatter diagram for Region B. **[1]**

c) There is no reason why the large island would have different transport methods to other local authorities. So it probably wouldn't appear as an outlier

14 A random variable X has probability distribution given by $P(X = x) = \frac{1}{860}(1 + x)$ for $x = 1, 2, 3, \ldots, 40$.

(a) Find $P(X > 39)$. **[2]**

14 a) $P(X > 39) = P(X = 40)$

$= \frac{1}{860}(1 + 40)$

$= \frac{41}{860}$

(b) Given that x is even, determine $P(X < 10)$. **[6]**

b) $P(X \text{ is even}) = P(X=2) + P(X=4) + \ldots + P(X=40)$

$= \frac{1}{860}(1+2) + \frac{1}{860}(1+4) + \cdots + \frac{1}{860}(1+40)$

$= \frac{1}{860}(3 + 5 + \cdots + 41)$

$\text{Sum} = \frac{n}{2}(a + l)$

$= \frac{1}{860}\left(\frac{20}{2}(3 + 41)\right)$

$= \frac{22}{43}$

$P(X \text{ is even and } X < 10) = P(X = 2, 4, 6, 8)$

$= \frac{1}{860}(3 + 5 + 7 + 9)$

$= \frac{6}{215}$

$P(X < 10 \mid X \text{ is even}) = \frac{P(X \text{ is even and } X < 10)}{P(X \text{ is even})}$

$= \frac{6}{215} \div \frac{22}{43}$

$= \frac{3}{55}$

END OF QUESTION PAPER